THERMODYNAMIC ANALYSIS OF COMPRESSED AIR VEHICLE PROPULSION

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Abstract

The fist compressed air vehicles were built by Andraud and Tessié du Motay in Paris between 1838 and 1840. Since then the idea has been tried again and again, but has never reached commercialization. In recent years the French developer MDI has demonstrated advanced compressed air vehicles. However, the claimed performance has been questioned by car manufacturers and automobile expert. Basically, when referred to ambient conditions, the relatively low energy content of the compressed air in a tank of acceptable volume is claimed to be insufficient to move even small cars over meaningful distances.

On the other hand, another air car developer claims to have driven 184 km on one 300 Litre filled with air at initially 300 bar pressure. Obviously, there are issues to be resolved, not by heated debates, but by an analysis of the thermodynamic processes involved. This is the aim of this study.

The results indicate that both sides are correct. At 20°C a 300 Litre tank filled with air at 300 bar carries 51 MJ of energy. Under ideal reversible isothermal conditions, this energy could be entirely converted to mechanical work. However, even under isentropic conditions (no heat is exchanged with the environment or generated by internal friction) not more than 25 MJ become useful. By multi-stage expansion with inter-stage heating the expansion process is brought closer to the isothermal ideal.

The analysis is extended to the compression of air. Again, the ideal isothermal compression is approached by multi-stage processes with inter-cooling. By this approach compression energy requirements are reduced to acceptable levels and system pressure and temperature are kept within safe limits.

The results of this analysis seem to indicate that the efficiency of the four-stage expansion process is acceptable, while even a four-stage air compression with inter-cooling is associated with significant losses. However, the overall energy utilization could be increased if the waste heat generated during the air compression process would be used for domestic water and space heating.

It seems that there is some justification for continuing the development of compressed air cars. However, it would be useful to establish the performance of such vehicles by an endurance race under controlled conditions in the presence of the general public.

1. Introduction

The following two questions need to be answered.

Question A

How much compression energy is needed to fill the tank with air at final pressure (300 bar = 30 MPa), but ambient temperature (20°C = 293.15K)?

The compression processes is treated as polytropic changes of state. The compression from the initial air volume V1 to the final tank volume $V_2 = V_3$ is followed by heat removal at constant tank volume V_3 from (p_2, T_2) to $(p_3, T_3 = T_1)$, i.e. back to the original ambient temperature T_1. The final conditions $(p_3, T_3 = T_1)$ can also be reached by an ideal isothermal compression from (p_1, V_1). The technical work input by isothermal compression W_{t13} is equal to the final energy content of the tank irrespective of the chosen path of polytropic compression and isochoric cooling. However, the energy consumption for compression is related to the polytropic efficiency. The energy input for a polytropic expansion is always higher than the energy input for an isentropic compression and much higher that the reversible

energy input under isothermal conditions.

Question B

How much mechanical energy can be recovered by expanding the compressed air in an air motor?

The expansion from $(p_3, T_3=T_1)$ to $(p_4=p_1, T_4<T_1)$ is also considered to be polytropic. For an ideal isothermal expansion the reversible isothermal technical work input W_{t13} can be recovered by the reversible process. However, in reality less energy is converted to technical work by a polytropic expansion process. An isentropic expansion would yield more technical work than the polytropic process, but much less than could be recovered by an ideal reversible isothermal expansion.

The air is exhausted at low temperatures after the extraction of expansion work. In an overall energy balance the heat taken from the ambient to restore initial air temperatures is less than the heat released during air compression because of non-ideal processes involved.

There are technical options for compression and expansion. The following analysis will suggest useful clues for the design of a compression-expansion system for air cars with acceptable driving performance that make good use of the electric energy needed to fill the tank with compressed air.
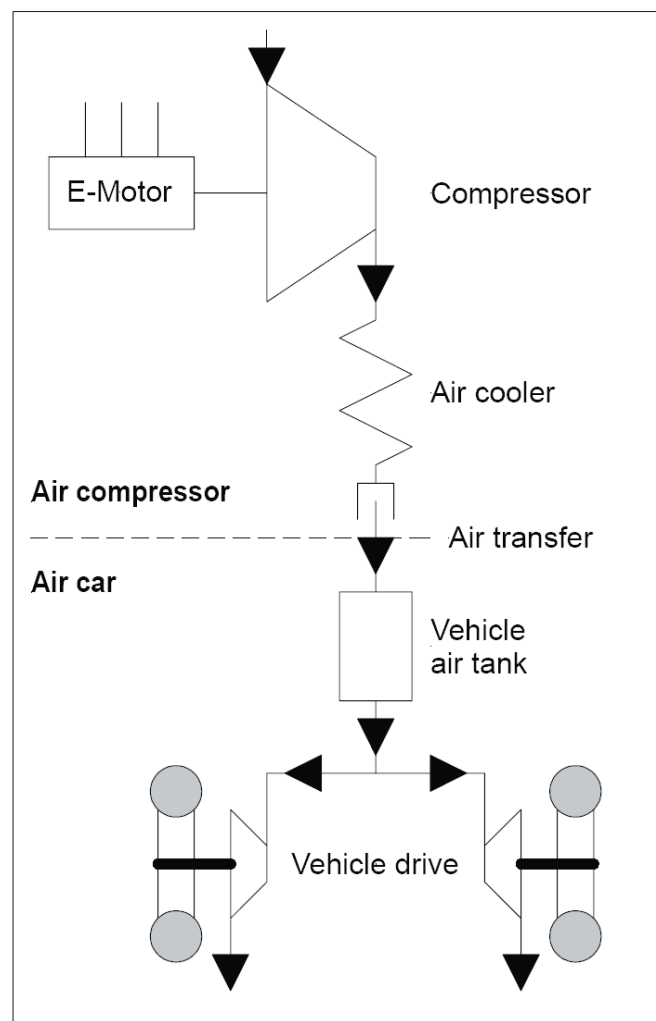

Fig. 1. Schematic of air compression, compressed air transfer to car and the use of compressed air for vehicle propulsion

2. Reference conditions

Reference conditions:

Normal pressure $p_0 = 760$ mmHg $= 1.01325$ bar $= 0.101325$ MPa

Normal temperature $T_0 = 0°C = 273.15$ K

Air density at NTP $\rho_0 = 1.2922$ kg/m^3

Initial conditions ("1"):

Ambient temperature $T_1 = 20°C = 293.15$K

Ambient pressure $p_1 = 1$ bar $= 0.1$ MPa

Air density $\rho_1 = 1.1883$ kg/m^3

Original air volume $V_1 = V_3 * p_3/p_1 = 90$ m^3 (before compression)

Mass of air $m_1 = V_1 * \rho1 = 106.95$ kg

Final conditions inside filled tank ("3"):

Tank volume $V_3 = 300$ Litre $= 0.3$ m^3

Air temperature $T_3 = T_1 = 20°C = 293.15$ K

Pressure in air tank $p_3 = 300$ bar $= 30$ MPa

Air density $\rho_3 = 356.49$ kg/m^3

Mass of compressed air $m_3 = V_3 * \rho_3 = 106.95$ kg $= m_1$ (check)

Final conditions after expansion ("4"):

Air pressure $p_4 = 1$ bar $= 0.1$ MPa

3. Air Compression

The considered compression processes are illustrated in a pressure-volume diagram (Figures 2) and a temperature-entropy diagram (Figure 3). Both presentations are commonly used for thermodynamic analyses.
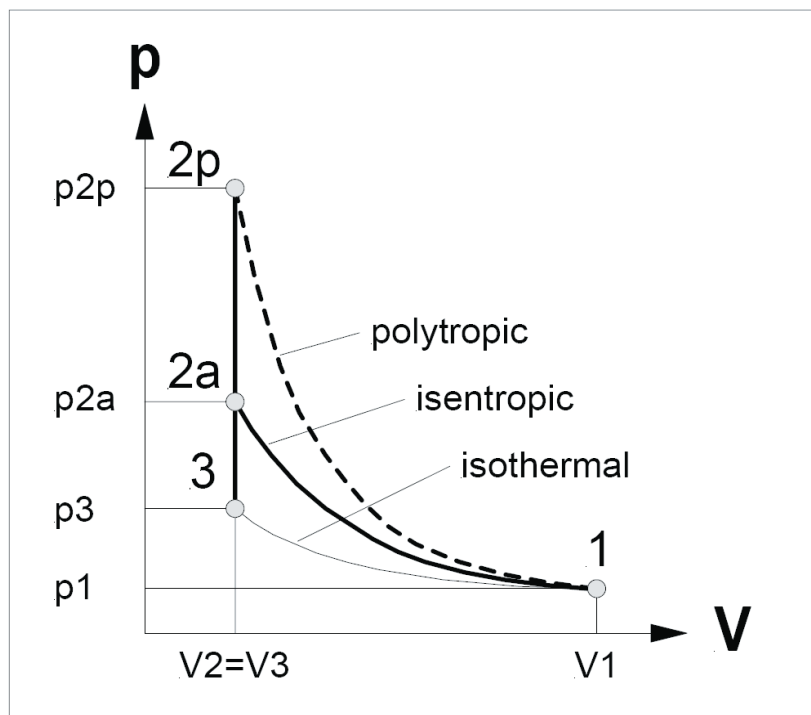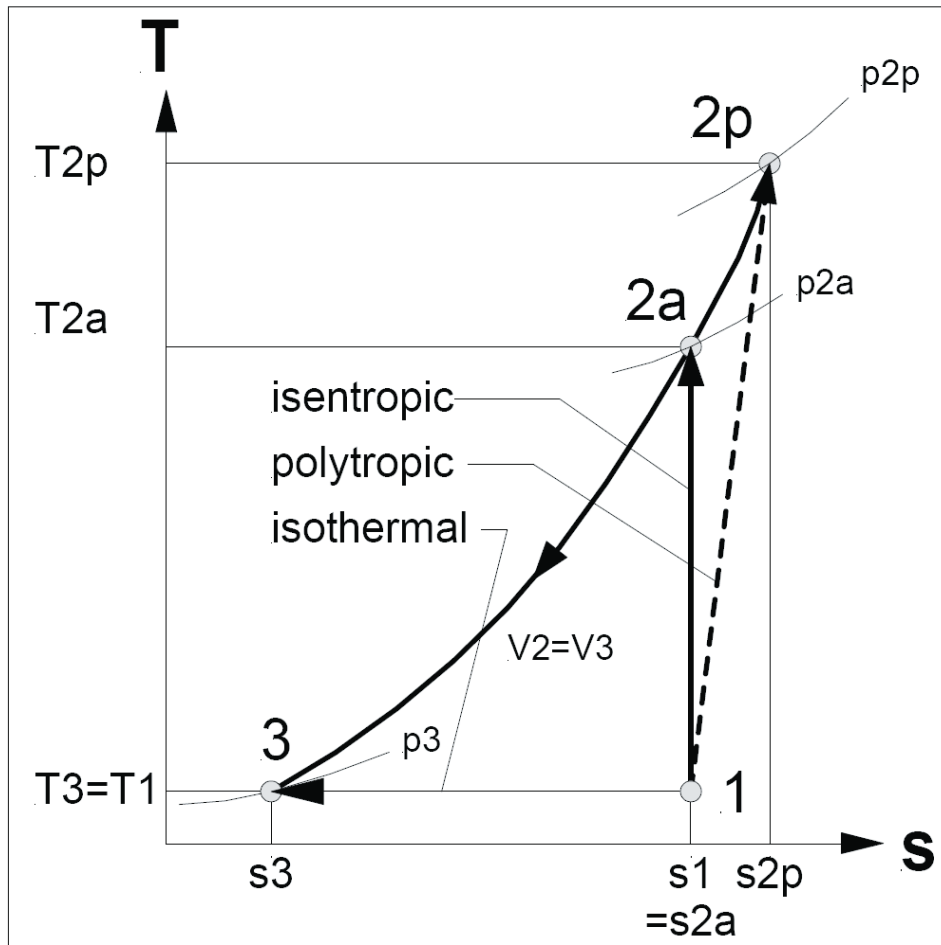

Fig. 2. P-V Diagram of single-stage air compression

Fig. 3. T-s Diagram of single-stage air compression

This is an idealized case. During the reversible compression process the temperature is considered to remain unchanged. The final temperature T3 is also the initial temperature T1. This requires that all heat is removed during the compression process by heat exchange with the environment (e.g. by transfer to a colder medium).

The technical work required for filling the tank with air from (p_1, T_1) to $(p_3, T_3=T_1)$ under isothermal conditions is

$$W_{t13} = W_{13} = p_1 * V_1 * \ln(p_3/p_1) = p_1 * V_1 * \ln(V_1/V_3). \tag{1}$$

4. Polytropic compression followed by isochoric cooling

The polytropic change of state follows the equation of an isentropic change of state, except that the isentropic coefficient γ (= 1.4 for air) is replaced by a polytropic coefficient n. The isentropic limit is obtained for n = 1.4 while n = 1 is valid for the isothermal case. Air is treated as an ideal gas.

In the isentropic case, no heat is leaving the system and no heat is generated by internal friction or poor aerodynamics, while in the polytropic case some heat is exchanged with the environment or available form internal aerodynamic losses. The isentropic case is the second idealized limit for real compression or expansion processes. However, a polytropic compression process is always associated with an increase of entropy as seen in the T-s-

Diagram (Figure 3).

The polytropic coefficient is related to the polytropic compression efficiency η_{com} of the process by

$$n = y / (y - 1/\eta_{com} * (y - 1)). \tag{2}$$

For $\eta_{com} = 0.9$ a polytropic compression coefficient of 1.47 is obtained.
For polytropic air compression from initial (p_1, V_1) to final (p_2, V_2) with $V_2 = V_3$, the technical work required is given by the following equation:

$$W_{t12} = m * c_p * (T_2 - T_1) = p_1 * V_1 * n/(n-1) * [(V_1/V_3)^{\wedge}(n-1) - 1]. \tag{3}$$

This leads to a significant temperature rise from (p_1, T_1) to (p_2, T_2). The intermediate pressure p_2 and temperature T_2 are obtained from

$$p_2 = p_1 * (V_1/V_3)^{\wedge}n, \tag{4}$$

$$T_2 = T_1 * (V_1/V_3)^{\wedge}[(n-1)/n]. \tag{5}$$

Finally, a thermodynamic efficiency of compression can be defined as the ratio of useful energy in the tank to the total technical work required to fill the tank with compressed air.

$$\eta_{th} = W_{t13} / W_{t12}. \tag{6}$$

The following significant results are obtained for different polytropic coefficients:

Table 1. Results for different polytropic coefficients

Single-stage compression	isothermal	isentropic	polyropic	polytropic	polytropic	polytropic	Units
Polytropic efficiency η_{com}	oo	1.0	0.95	0.90	0.85	0.80	-
Polytropic coefficient n	1.0	1.4	1.43	1.47	1.51	1.56	-
Technical Work W_{t12}	51	277	318	374	454	574	MJ
Energy in tank W_{t13}	51	51	51	51	51	51	MJ
Efficiency W_{t13}/W_{t12}	1.00	0.19	0.16	0.14	0.11	0.09	-
	100	19	16	14	11	9	%
Final temperature T_2	20	2,597	3,135	3,888	4,991	6,697	°C

The results clearly indicate that the compression has to proceed close to the isothermal limit. Anything else would not only lead to excessive temperatures T2 and pressures p2, but also to high energy consumption Wt12 for air compression. This can be accomplished only with multi-stage compression with inter-cooling.

Acceptable compression efficiency is obtained only, if the process is designed to proceed close to the isothermal limit, i.e. at gas temperatures near ambient air temperature. However, compressing air in a single stage compressor even at high polytropic efficiency is totally

unsatisfactory with respect to energy losses, temperature and pressure. The analysis does not include real gas effects which make things even worse. Multi-stage compressors with heat exchangers between successive compression stages provide better solutions.

5. Four-stage compression

A four-stage compression process is now analyzed. Heat is removed between stages by three intercoolers and between the final stage and the tank. The polytropic compression process is started with air under normal conditions (p_1 = 1bar, T_1 = 20°C). For all following stages the inlet air temperature is assumed to be 20°C as well. In practice, this can only be accomplished by heat transfer to cold media, i.e. on cold days when the ambient air temperature is sufficiently low.

The four-stage compression process is sketched in the temperature-entropy presentation of Figure 4. Compared to the single-stage process (point 2) excessive air temperatures are avoided by multi-stage compression. Also, the compression work needed is reduced significantly as shown by the tabulated results below.
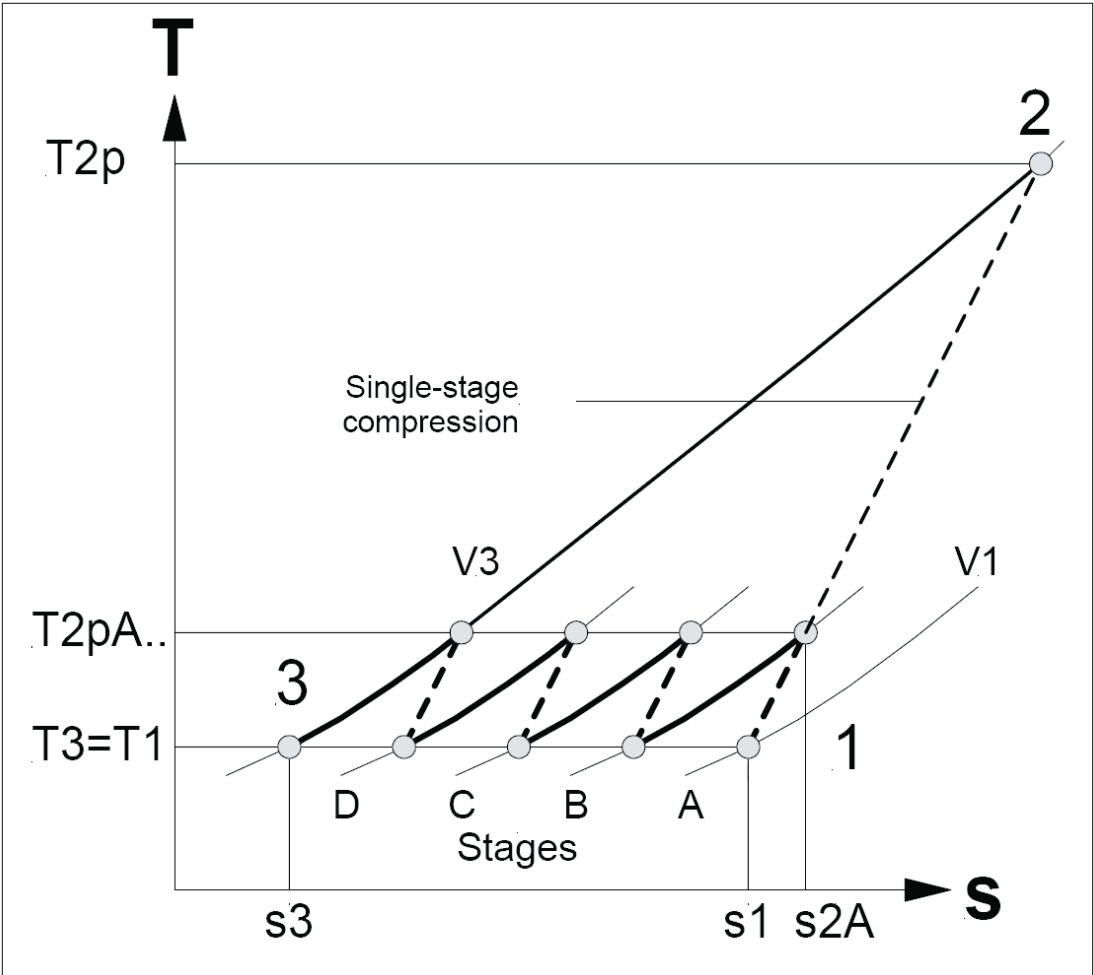

Fig. 4. T-s-presentation of a four-stage compression compared to a single-stage compression (point 2)

For the simplified analysis each of the four stages was assumed to operate at the same compression ratio of 4.162, equal to the 4th root of the total pressure ratio of 300. For a polytropic compression efficiency of 0.9 and a corresponding polytropic exponent n = 1.47 the following results are obtained:

Table 2. Results for compression efficiency of 0.9 and a corresponding polytropic exponent n = 1.47

Stage identification	A	B	C	D	Units
Inlet pressure	0.1	0.42	1.73	7.21	MPa
Outlet pressure	0.42	1.73	7.21	30.00	MPa
Inlet temperature	20	20	20	20	°C
Outlet temperature	296	296	296	296	°C
Inlet air volume (20°C)	90	21.63	5.20	1.25	m3
Outlet air volume	21.63	5.20	1.25	0.30	m3
Inlet air density	1.19	4.95	20.58	85.66	kg/m3
Outlet air density	4.95	18.67	77.71	323.39	kg/m3

The equations (1) to (6) derived for a single stage air compression are used for each stage of the four subsequent processes. The overall results are obtained by summing the technical work input or the heat released of each stage. The results strongly depend on the polytropic coefficient n, i.e. on the polytropic efficiency or the aerodynamic quality of the compressor.

Table 3. The overall results obtained by summing the technical work input or the heat released of each stage

Four-stage compression	isothermal	isentropic	polytropic	polytropic	polytropic	polytropic	Units
Polytropic efficiency η_{com}	∞	1.0	0.95	0.90	0.85	0.80	-
Polytropic coefficient n	1.0	1.4	1.43	1.47	1.51	1.56	-
Compression work W_{12}	51	77	101	107	113	122	MJ
Energy in tank W_{13}	51	51	51	51	51	51	MJ
Efficiency W_{13}/W_{12}	1.00	0.66	0.51	0.48	0.45	0.42	-
	100	66	51	48	45	42	%
Final temperature T_2	20	150	269	296	330	374	°C

Obviously, significant energetic and thermodynamic advantages can be obtained when the compression process is spread over multiple stages with integrated inter-cooling. Even for an isentropic compression process the overall thermodynamic efficiency is increased from 16% to 53% by staging. Realistic may be a polytropic compression with $\eta_{com} = 95\%$ and n = 1.43 yielding an isothermal efficiency of 51% for the four-stage process. For a four-stage compression with air inter-cooling to ambient temperature of 20°C the maximum isothermal efficiency of 66% can be obtained under isentropic conditions. Even higher values may be obtained by adding compression stages or by cooling the air between stages to temperatures below ambient. However, such units are voluminous, heavy and expensive. They may not be suited for onboard applications in vehicles.

6. Technical work derived from the stored pressurized air

Technical work is recovered by expanding the compressed air in suitable expansion engines from tank conditions ($p_3 = 30$ MPa, $T_3 = 20°C = 293K$) to ambient pressure ($p_4 = 0.1$ MPa = 1 bar). In the process the temperature of the expanding air will drop to very low levels depending on the thermodynamic quality of the expansion. The lowest temperatures are

reached for an isentropic expansion when no heat is exchanged with the environment. The other extreme is the isothermal expansion at constant temperature.

With respect to ambient temperature $T_1 = 20°C$ the energy content of the tank is equal to the isothermal technical work W_{t13}. This energy can be recovered only partially by an isentropic or polytropic expansions as indicated by the area formed by the expansion curves and the ordinate, i.e. by the p*dV integral.
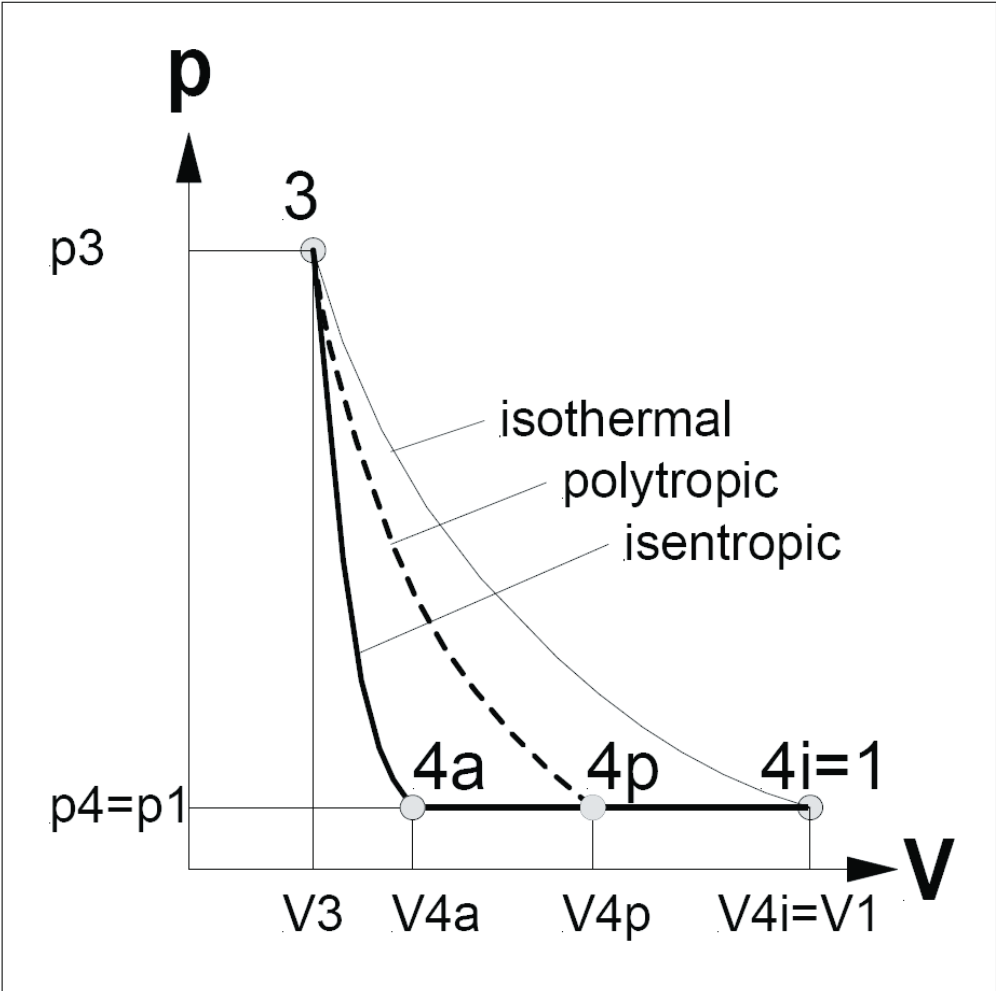

Fig. 5 p-V-diagram of single-stage expansion processes

7. Single-stage expansion

The thermodynamic equations presented above for air compression are also valid for air expansion. The polytropic coefficient is related to the polytropic efficiency ηexp of the process by

$$n = γ / (γ − η_{exp} * (y - 1)). \tag{7}$$

This equation for the polytropic expansion coefficient is similar to equation (2) for the polytropic compression coefficient with one exception. In de denominator the polytropic efficiency ηexp appears as factor, not as reciprocal value. As a result, the values of the polytropic expansion coefficients are always less than the isentropic number of 1.4. A polytropic expansion coefficient of 1.35 is obtained for a polytropic expansion efficiency of ηexp = 0.9.

For a generalized polytropic expansion from the initial conditions (P_3 = 30 MPa and T_3 = 20°C = 293K) to p_4 = 0.1 MPa = 1 bar with ($p_3V_3 = p_1V_1$) the technical work recovered is given by

$$W_{t34} = m * c_p * (T_4 - T_3) = p_1 * V_1 * n/(n-1) * [(p_4/p_3)^{\wedge}(n/(n-1)) - 1]. \qquad (8)$$
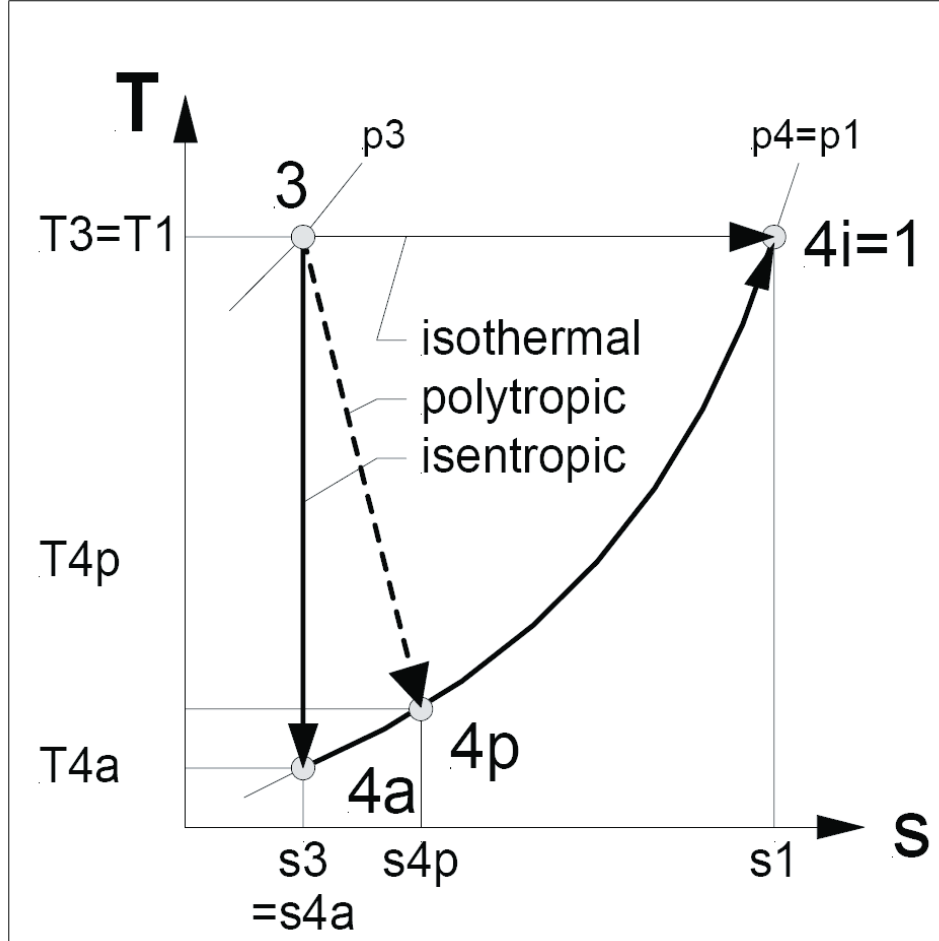

Fig. 6. T-S-diagram of single-stage expansion processes

The final temperature T_4 is obtained from

$$T_4 = T_3 * (p_4/p_3)^{\wedge}[(n-1)/n]. \qquad (9)$$

Finally, the total technical work efficiency is of interest. The useful technical work output of the expansion engine W_{t34} is related to the technical work input for compression W_{t12}:

$$\eta_{th} = W_{t34} / W_{t12}. \qquad (10)$$

For a single-stage expansion the following results are obtained.
The following results represent a polytropic expansion of air from tank conditions p_3 = 30 MPa and T_3 = 20°C. All equation for work or energy yield negative results as work is extracted from the system. However, minus signs are omitted from the tabulated numbers.

Table 4. Results represent a polytropic expansion of air from tank conditions p_3 = 30 MPa and T_3 = 20°C

Single-stage compression	isohermal	isentropic	polytropic	polytropic	polytropic	polytropic	Units
Polytropic efficiency ηexp	0	1.0	0.95	0.90	0.85	0.80	-
Polytropic coefficient n	1.0	1.40	1.37	1.35	1.32	1.30	-
Technical Work Wt34	51	25	26	27	28	29	MJ
Energy in tank Wt13	51	51	51	51	51	51	MJ
Efficiency Wt34 / Wt13	1.0	0.49	0.51	0.52	0.54	0.56	'-
	100	49	51	52	54	56	%
Final temperature T4	20	-216	-211	-206	-200	-194	°C

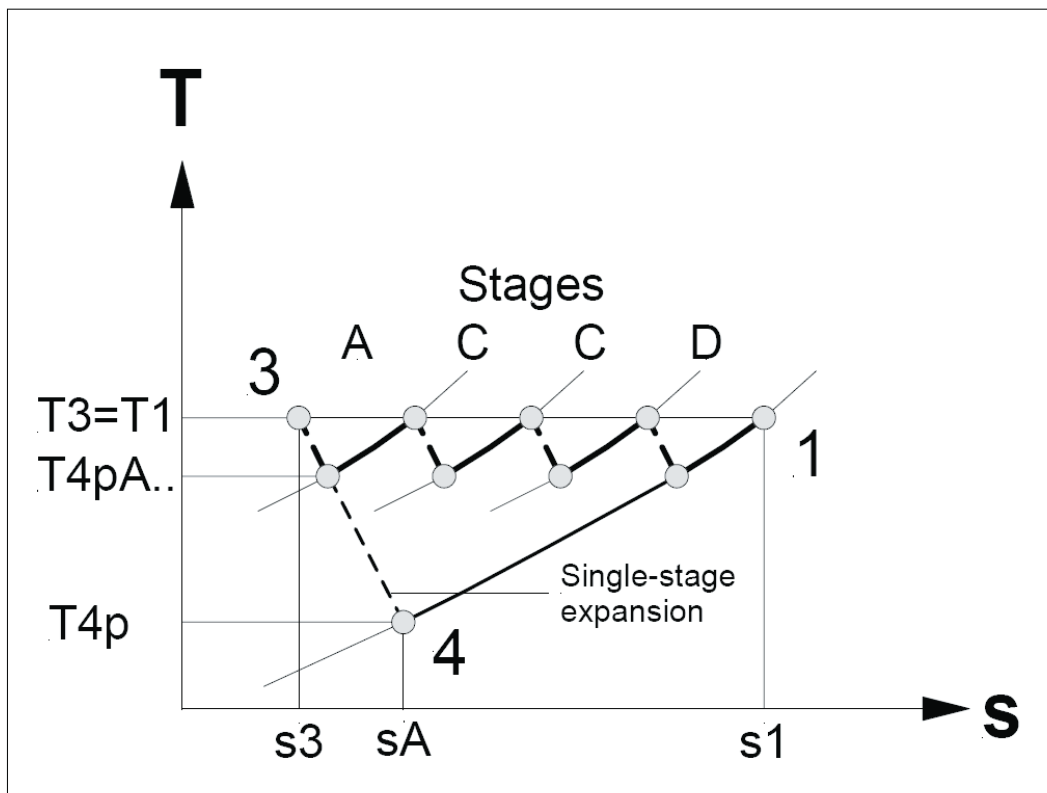*Fig. 7. T-s-diagram of a four-stage expansion compared to a single-stage expansion (point 4)*

In the isentropic case the theoretical final temperature of the expanded air drops to minus 216°C, i.e. well below the temperature of liquid air at atmospheric pressure. Needless to say, this does not represent reality. However, it indicates that technical problems may limit the extraction of mechanical work from compressed air by expansion engines.

8. Four-stage expansion

The lowest technical work Wt_{34} is recovered from an isentropic expansion. For polytropic expansions the temperature of the air is raised by heat generated during the process as a result of friction or non-ideal aerodynamics, or heat is accepted from the environment by heat

exchange. As expected, the highest efficiency is obtained when the expansion process proceeds close to the isothermal limit. From an energetic standpoint, the best results may be obtained by using a multistage expansion motor with heating between stages.

The thermodynamics of a four-stage expansion with three heat exchangers has been analyzed. As the cold exhaust is released into the atmosphere, nature will take care of the final heat exchange to restore the original ambient conditions. It is assumed that all heat exchangers are sized to raise the temperature of the air exhaust of all but the last stage to 20°C. This is not easily accomplished on hot summer days, but may be possible under cold weather conditions. Again, pressure ratios equal to the 4th root of the overall expansion ratio of 300 are assumed for the expansion process.

The numbers in the following table represent a polytropic expansion efficiency of $\eta_{exp} = 0.9$, or a polytropic exponent of n = 1.35.

Table 4. Polytropic expansion efficiency of $\eta_{exp} = 0.9$, or a polytropic exponent of n = 1.35.

Stage identification	A	B	C	D	Units
Inlet pressure	30	7.21	1.73	0.42	MPa
Outlet pressure	7.21	1.73	0.41	0.10	MPa
Inlet temperature	20	20	20	20	°C
Outlet temperature	- 70	- 70	- 70	- 70	°C
Inlet air volume (20°C)	0.30	1.25	5.20	21.63	m3
Outlet air volume	1.25	5.20	21.63	90.00	m3
Inlet air density	356.49	91.93	22.09	5.31	kg/m3
Outlet air density	119.04	30.70	7.38	1.77	kg/m3

Similar parameters can be obtained for other polytropic expansion efficiencies ηexp. In the following table results are presented for the stated inter-heating assumptions, but five different polytropic conditions. The listed results are obtained by summing the technical work output of all four stages. The results strongly depend on the choice of polytropic coefficient n as shown in the following table. Again, the minus signs are omitted for the extracted work.

Table 5. Results obtained by summing the technical work output of all four stages

Four-stage expansion	iso-thermal	isen-tropic	poly-tropic	poly-tropic	poly-tropic	poly-tropic	Units
Polytropic efficiency ηexp	∞	1.0	0.95	0.90	0.85	0.80	-
Polytropic coefficient n	1.0	1.4	1.37	1.35	1.32	1.30	-
Expansion work Wt34	51	42	43	43	43	44	MJ
Energy in tank Wt13	51	51	51	51	51	51	MJ
Efficiency Wt34/W113	1.00	0.82	0.83	0.84	0.85	0.85	-
	100	82	83	84	85	85	%
Final temperature T4	20	-78	-74	-70	-66	-62	°C

7. Overall Efficiency

The overall efficiency of the air car is defined as the useful technical work Wt34 obtained from expanding the compressed air compared to the energy Wt12 needed to fill the tank. For single-stage compression and expansion this ratio is unacceptably low. But it becomes reasonable for multi-stage processes.

For the sample case of identical polytropic efficiencies $\eta_{com} = \eta_{exp} = 0.9$ for compression and expansion and for four-stage processes the overall efficiency becomes

$$\eta_{tot} = Wt34 / Wt12 = 42.97 \text{ MJ} / 106.72 \text{ MJ} = 0.4027 = 40.27\%. \qquad (11)$$

This efficiency does not include electrical and mechanical losses and parasitic power consumption related to the transfer of electrical energy to compression energy of the stored air, or similar losses related to the conversion of the pressure energy into vehicle motion by means of suitable expansion engines.

8. Conclusions

For the operation of a compressed air car the overall "plug-to-road" efficiency is one of the key criteria. The optimum is obtained when maximum technical work Wt34 becomes available at a minimum of technical work W_{t12} input for air compression. From the foregoing analysis it becomes clear that both compression and expansion must proceed close to the isothermal limit. This can only be accomplished with multi-stage compression and expansion processes with heat exchangers for removal or addition of heat to the medium to establish close to ambient conditions.

The foregoing analysis may not be the first of its kind and certainly needs refinements. In particular, the thermodynamics of heat exchange, mechanical and aerodynamic losses, electrical efficiencies etc. need to be considered. All these effects may reduce the overall efficiency to 40% or less. The total process efficiency may be improved by increasing the number of compression and expansion stages. However, such efficiencies may still be attractive in a sustainable energy future when renewable energy is harvested as electricity and transportation needs must be satisfied from available energy sources. With respect to overall efficiency, battery-electric vehicles may be better than air cars, but hydrogen fuel cell systems may be worse. However, with respect to system and operating costs, air cars may offer many advantages such as simplicity, cost, independence, zero pollution and environmental friendliness of all system components.

All in all, the compressed air car seems to be a viable option for clean and efficient short range transportation. Further analyses, additional research and development are most welcome to fully identify the potentials of this unconventional source of transportation energy.